**Finding the “Best” Second Order Regression Model in a Polynomial Number of Steps**

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The number of all possible regression models, allowing for interaction and curvature, grows exponentially with the number of explanatory variables. Fitting all possible models and choosing the best one has, therefore, been considered impractical. We consider two stepwise model selection approaches, stepAIC and Greedy (a new algorithm), that decrease computational effort by considering only a small subset of all possible models. We show that Greedy considers a number of models quadratic in the number of explanatory variables, whereas stepAIC considers a number of models cubic in the number of explanatory variables. In the examples given, and in the experience of two of the authors, these methods identify a model that is either “best” or nearly best based on criteria such as AIC.

KEY WORDS: Curvature, Greedy algorithm, Interaction, Model building, Model selection, stepAIC.

1. **INTRODUCTION**

Many would consider a model building exercise that considers all possible second order regression models a daunting task. The complete second order model (sometimes called the saturated second order model) contains all main effects, a curvature term (squared term) for all main effects, and all possible two-way interactions.

For example, for a model with three explanatory variables this would be:

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Considering this model, and all valid reduced models, is indeed involved. We define a valid reduced model to include any subset of variables in the complete second order model, with the usual caveat that if an interaction or curvature term is in the subset model, then the associated main effect must also be included. Therefore is a valid reduced model, but is not.

The number of possible reduced models grows exponentially with the number of explanatory variables. Specifically, let be the number of valid second order models, when the number of explanatory variables is . Then The two most popular stepwise model selection algorithms within the R statistical environment (R development core team 2016), step() and stepAIC() (Venables and Ripley, 2002), fit about total models (see Appendix 1). Conversely, an algorithm we introduce here, that we denote “Greedy” fits models. Table 1 contrasts the number of models fitted by these polynomial time algorithms versus the number of models that would need to be fitted to compare all possible second order models.

*Table 1. Models considered versus all possible models*

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

|  |  |  |  |
| --- | --- | --- | --- |
| *Explanatory variables* | *Greedy* | *stepAIC* | *= number of valid models* |

|  |  |  |  |
| --- | --- | --- | --- |
| 0 | 1 | 1 | 1 |
| 1 | 3 | 3 | 3 |
| 2 | 6 | 10 |  |
| 3 | 10 | 27 | 5 |
| 4 | 15 | 54 |  |
| 5 | 21 | 96 |  |
| 6 | 28 | 153 |  |

1. **AUTOMATED MODEL SELECTION**

Most variable selection procedures, as advocated in textbooks, appear to have difficulties when including interaction and/or curvature terms. Those same textbooks suggest that consideration of these rich models is an overwhelming task.

Mendenhall and Sincich (2012) write, after an extensive presentation of stepwise regression and all-possible-regressions: “…analysts typically do not include higher-order terms or interactions in the list of potential predictors for stepwise regression. Therefore, if no model building is performed, the final model will be a first-order, main effects model. “ and “even if the analyst includes some higher-order terms and interactions as potential predictors, the stepwise and best subsets procedures will more than likely select a nonsensical model “. On the other hand, the authors note: “Most real-world relationships between variables are not linear, and these relationships often are moderated by another variable (i.e. interaction exists).”

The much venerated (at least in our eyes), Ramsey and Schafer (2002), in their chapter on variable selection, write: “The SAT study featured *k*=6 explanatory variables. The SSOM (saturated second order model) for that problem contains 28 parameters. …and the total of all hierarchical models to consider is 2,104,489. Things quickly get out of hand, and it becomes necessary to plan a strategy for sorting through some-but not all- of the models for promising candidates.” The authors go on to suggest a bewildering variety of ad hoc approaches for further exploration of the data for better models.

Montgomery, Peck and Vinning (2012) and Weisberg (2014) simply ignore interaction and curvature in their chapters on variable selection and discuss only models with main effects. It should be noted that the more carefully a textbook discusses curvature and/or interaction in the earlier chapters of the book, the more glaring the omission of these ideas in chapters on variable selection.

Clearly second order model selection is complicated. We need to include curvature and interaction terms, but the possible models grow so quickly that exhaustive consideration of all valid reduced models is impractical.

1. **STEPAIC AND GREEDY**

StepAIC has several directional options including ”forward”, “backward” and “both.” We describe the backward selection version of the algorithm. This is the default method if a “scope” argument is not specified.

The stepAIC algorithm:

1. Form a very rich model, often the complete second order model
2. Consider all variables currently eligible to be dropped (main effects cannot be dropped in associated interaction and/or curvature terms are in the model), fit each associated model and compute the AIC value for that model.
3. Drop the variable with associated model having the lowest AIC value.

Repeat until all variables have been dropped.

The model with the best score is chosen.

Two of the authors have, in practice, often used a simpler approach, given by the algorithm below. In fact, this method is probably in wide use by practitioners, although it seems to have never impacted textbook writers.

The Greedy algorithm:

1. Form a very rich model, often the complete second order model. Assign a score to this model.
2. Pick a variable to drop, from the list of eligible variables (as defined above). The eligible variable with the smallest *t*-statistic (in magnitude) is chosen.
3. Drop that variable and assign a score to the new model

Repeat until all variables have been dropped.

The model with the best score is chosen.

It should be noted that for small model selection problems with limited explanatory variables, these algorithms inform and guide model selection, but in this case model selection is not strictly algorithmic.

Typical scoring of models includes Akaike information criteria (Akaike, 1974), Bayes information criteria (Schwarz, 1978*)*, *Mallows’s Cp* (Mallows, 1973*)*, or *PRESS* (Allen, 1974) in the form of . These will be referenced respectively as *AIC*, *BIC*, *Cp*, and *PRESS* below.

StepAIC is more “careful” than Greedy concerning which variable is dropped at each step, but fits more models. It happens that Greedy is quadratic in the number of models fitted, while stepAIC is cubic in the number of models fitted (see Appendix 1).

1. **TWO EXAMPLES**

Two examples from Ramsey and Schafer (2002) were chosen, each with three explanatory variables. Thus, there are 95 valid models (Table 1). Four common scoring criteria, mentioned above, are considered for Greedy. Our focus will be on Greedy, but stepAIC and Greedy, using AIC as the score, correctly identified the same best model in both cases.

* 1. **Example 1**

Case 2 from Chapter 9 provides a power law model for brain size and body size for 96 mammal species. In predicting brain size on the log scale, the three variables considered are body size on the log scale, litter size, and gestation period:

, , , .

Table 2 shows the results when Greedy is applied to this data, including all steps. The algorithm involves nine steps and considers ten models. However, the single mean model, which terminates the algorithm, is typically so poor that it is of no interest. In the case of *AIC*, *Cp* and *PRESS*, the algorithm found the best model as confirmed by an examination of all 95 models. For *AIC* the best model found by Greedy had a value of 123.0 while the *best possible* *BIC* value was 143.5 (Table 2). The best model, according to both *AIC* and *BIC*, was ,.

*Table 2. The nine steps in the algorithm, example 1. Best value of each criteria underlined.*

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *Model* | *AIC* | *BIC* | *PRESS* | *Cp* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| , | 126.6 | 154.8 | 93.8% | 10.0 |
| Drop :  , | 124.7 | 150.3 | 93.8% | 8.11 |
| Drop :  , | 123.3 | 146.4 | 94.8% | 6.70 |
| Drop :  , | 123.0 | 143.5 | 95.1% | 6.20 |
| Drop :  , | 126.2 | 144.1 | 95.5% | 9.11 |
| Drop:  , | 131.1 | 146.4 | 95.1% | 14.01 |
| Drop :  , | 136.4 | 149.2 | 94.4% | 19.92 |
| Drop : | 159.1 | 169.3 | 93.5% | 49.60 |
| Drop : | 171.2 | 178.9 | 92.7% | 69.71 |

The *PRESS* criterion suggests the optimality of a slightly less complex model. *PRESS* is different from the other three criteria in that it considers influential points through the use of “hat” values. It is likely the preference given by *PRESS* is based on the role of leverage points in the models examined.

It is also worth examining the range of possible values for each criteria, for the models considered. Table 3 gives a five number summary for the range of values possible. Given that a difference of 6 is quite large for information criteria (Burnham and Anderson, 2002; Kass and Raftery, 1995), most models are much worse that the selected model.

*Table 3. Range of possible values, example 1*

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *Criteria* | *best* | *Q1* | *median* | *Q3* | *worst* |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *AIC* | 123.0 | 124.7 | 126.6 | 136.4 | 171.2 |  |
| *BIC* | 143.5 | 146.4 | 149.2 | 154.8 | 178.9 |  |
| *Cp* | 6.20 | 11.82 | 16.16 | 27.45 | 1345.0 |  |
| *PRESS* | 95.5% | 94.9% | 94.6% | 93.6% | 33.2% |  |

**4.2 Example 2**

Case 2 from Chapter 12 in Ramsey and Schafer (2002) involves an observational study exploring factors impacting salary.

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There were only 52 possible models because squaring the sex indicator variable does not create a new variable (it could be argued this is not the complete second order model, but it is a reasonable starting point).

Table 4 shows the results of the greedy algorithm applied to this data. In this exercise, the best model was found from the perspective of all four criteria, although different scoring methods found different best models. *AIC* and *Cp* found very complex models to be optimal, whereas *BIC* viewed a relatively simple model as best (*BIC* has a more severe penalty for complexity than *AIC*).

*Table 4. The nine steps in the algorithm, example 2. Best value of each criteria underlined.*

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *Model* | *AIC* | *BIC* | *PRESS* | *Cp* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1427.6 | 1452.9 | 45.6% | 8.00 |
| drop : | 1425.6 | 1448.4 | 47.6% | 6.00 |
| drop : | 1423.7 | 1444.0 | 49.3% | 4.12 |
| drop : | 1424.8 | 1442.5 | 50.2% | 4.90 |
| drop : | 1427.1 | 1442.3 | 48.3% | 7.00 |
| drop: | 1433.6 | 1446.3 | 44.5% | 13.58 |
| drop : | 1455.0 | 1465.1 | 29.8% | 39.75 |
| drop : | 1456.4 | 1463.9 | 29.3% | 42.42 |

For completeness Table 5 is presented, showing the range of values for each criteria for this example.

*Table 5. Range of possible values, example 2*

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *Criteria* | *best* | *Q1* | *median* | *Q3* | *worst* |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *AIC* | 1424 | 1425 | 1427 | 1439 | 1456 |
| *BIC* | 1442 | 1444 | 1447 | 1456 | 1465 |
| *Cp* | 4.12 | 8.63 | 27.81 | 38.37 | 94.37 |
| *PRESS* | 50.2% | 45.3% | 34.8% | 29.9% | 1.7% |

**5. COMPUTATIONAL TIME RUN-TIME COMAPRISONS FOR GREEDY AND STEPAIC**

As a computational proof of concept, we developed an automated version of the Greedy algorithm using the R language.

**5.1 Example 3**

The functions step and stepAIC identified the same “best” (low AIC) model as the Greedy algorithm for the case studies given in examples 1 and 2, with very similar run times. To compare computational run times for the Greedy automated algorithm and stepAIC in more complex model selection situations, we considered three datasets frequently used for testing data mining and machine learning applications, and a high dimensional randomly generated dataset. The datasets were: 1) concrete, a 1030 observation, eight predictor dataset for modelling concrete compressive strength (Yeh 1998 ), 2) facebook, a 500 observation twelve predictor dataset used by Moro et al (2016) to assess Facebook® performance metrics, 3) wine, a 4898 observation, eleven predictor dataset concerning the relationship of wine quality and physicochemical properties (Cortez et al. 2009 ), and 4) rmvm, a randomly generated 500 observation, fifteen predictor dataset obtained from a multivariate normal distribution with an identity covariance matrix. The rmvm response variable was defined to be a linear function of the predictors, plus standard normal errors. (Data sets will be available in CRAN and identified further in a non-blinded version of the paper) For each dataset, 15 model selection simulations were run for models with one (intercept) parameter, up to models with *k* predictors. For each simulation, explanatory variables were defined to be random subsets of the total *k* predictors. Accumulated computational time was measured in R using the same Intel® Core™ i7 CPU workstation with 12 GB RAM.

Fig. 1 clearly demonstrates that stepAIC is much slower than Greedy in selecting from model subsets considering all possible quadratic and interaction terms. This is particularly true for the wine dataset, which contained a large number of observations (Fig. 1c), and the rmvm dataset, which had the largest number of predictors (Fig. 1d). Greedy and stepAIC identified the same “best” model for the concrete data. The best stepAIC model had a slightly smaller AIC value than the best Greedy model for the facebook ΔAIC = 2.8 and wine ΔAIC = 2.9 datasets. Conversely, the best Greedy model had a slightly smaller AIC value for the rmvm dataset: ΔAIC = 0.351. Burnham and Anderson (2002) and others have noted that ΔAIC values of less than approximately 2 represent models with essentially identical parsimony.



Fig. 1. Computational run times for an automated Greedy function (Appendix 2) and stepAIC for randomized subsets of four datasets: a) concrete, b) facebook, c) wine, and d) rmvm.

1. **CONCLUDING REMARKS**

Both stepAIC and Greedy are greedy algorithms in the sense that they make local optimal steps, hoping to achieve (something close to) a global optimum. Greedy takes a less tortuous computational path through the solution space, and completes the algorithm fitting a number of model quadratic in the number of explanatory variable. StepAIC is more careful at each step, but requires a cubic number fitted models. Recall the only known guaranteed method of finding the best model is to examine all models, which requires fitting a number of model exponential in the number of explanatory variables.

Although any further evidence is anecdotal, we can say that when we have used a Greedy approach in the past extensive further exploration of models rarely finds a better model then the one initially produced by the algorithm. Although it is assumed hundreds of applied statisticians have used this approach, or something similar, on thousands of data sets, we cannot find mention of the algorithm in any standard textbook on regression. If nothing else can be concluded, it is that regression textbooks need to completely rethink their chapter on variable selection based on the existence of stepAIC and Greedy.

There are three clear follow up objectives. First, we are currently developing an optimized version of the automated Greedy function that better incorporates categorical variables and allows consideration of non-general linear models. Second we plan on proceeding with an in depth comparison of the relative merits of Greedy and other model selection approaches and assess the value of using these algorithms on very large data sets with varying degrees of collinearity. Third, there are algorithms even less computationally intensive than Greedy and hybrid algorithms that fall between stepAIC and Greedy in complexity. For really large data sets and different degrees of collinearity, algorithm variants may be useful.

**APPENDIX 1**

Given a number of explanatory variables, , and a complete second order model, the algorithm Greedy fits a number of models quadratic in . StepAIC, using a relatively efficient path through the algorithm, fits a number of models cubic in .

Let denote the total number of models fitted with Greedy. The complete second order model has main effects, quadratic terms and “ choose 2” interactions. The algorithm drops one variable at each step and never adds a variable. One model is fitted at each step. The sum of these terms equals the total number of steps and the total number of models fitted: .

Let denote the total number of models fitted using stepAIC. StepAIC, at each step, fits each model that could occur by dropping any variable eligible to be dropped. It then chooses which variable to be dropped, and repeats the process. Beginning with a complete second order model with explanatory variables, there is no unique path from the beginning to the end of the algorithm. However, one path that would require a small number of fitted models would be to drop a set of variables in the following order: . In other words, drop two quadratic terms, then the interaction, and finally the two main effects associated with the quadratic terms and interaction already dropped. This would reduce to a complete second order model with explanatory variables.

This path has relatively few fitted models because whenever you drop a number of quadratic terms and/or interactions, main effects slowly become eligible to be dropped. Dropping those main effects as soon as possible reduces the total number of models to be considered and fitted at each step. We do not claim, but suspect, this would produce the minimum number of fitted models for stepAIC.

This indicates 5 steps in a reduction from a complete second order model with explanatory variables to a complete second order model with explanatory variables. In any complete second order model, all quadratic terms, all interaction terms, but no main effects are eligible to be dropped.

|  |  |  |  |
| --- | --- | --- | --- |
| Step | Models fitted | Drop variable | Comment |
| 1 |  |  | Consider all interactions and quadratic terms |
| 2 |  |  | Consider all interactions and quadratic terms |
| 3 |  |  | Consider all interactions and quadratic terms |
| 4 |  |  | Consider interactions, quadratics and two main effects |
| 5 |  |  | Consider interactions, quadratics, and one main effect |

At this point we have a complete second order model with explanatory variables. Notice that at step 4 the three second order terms have been dropped, but two main effects have become eligible to be dropped. The number of total fitted models for stepAIC can be described using a recursive relationship.

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This immediately produces the difference We also know, by enumeration, that and .

Using differencing techniques, it is not hard to show , where when is odd and when is even.

**REFERENCES**

Allen, D. M. (1974), “The Relationship Between Variable Selection and Data Augmentation and a Method for Prediction,” *Technometrics*, 16, 125–127.

[Akaike, H.](https://en.wikipedia.org/wiki/Hirotugu_Akaike) (1974), [“A New Look at the Statistical Model Identification,](http://www.unt.edu/rss/class/Jon/MiscDocs/Akaike_1974.pdf)“ IEEE Transactions on Automatic Control, 19, 716–723.

Breiman, L. (2001),” Statistical Modeling: The Two Cultures, ” Statistical Science, 16, 199-215.

Burnham, K. P., and Anderson, D. R. (2002), Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach (2nd ed.), New York: Springer-Verlag.

Cortez, A. Cerdeira, F. Almeida, Matos, T., Reis, J. (2009), “Modeling Wine preferences by data mining from physicochemical properties,” *Decision Support Systems*, 47(4):547-553.

Kass, R. E., and Raftery, A. E. (1995), “Bayes Factors,” Journal of the American Statistical Association, 90, 773-795.

Mallows, C. L. (1973), "Some Comments on CP," Technometrics 15, 661–675.

Mendenhall, W., and Sincich, T. (2012), *A Second Course in Statistics* (7th ed.), New Jersey: Pearson.

Montgomery, D. C., Peck, E. A., and Vining, G.G. (2012), *Introduction to Linear Regression Analysis* (5th edition), New Jersey: Wiley.

Moro, S., Rita, P. and Vala, B. (2016), “Predicting social media performance metrics and evaluation of the impact on brand building: A data mining approach”, *Journal of Business Research,* 69(9): 3341–3351.

Ramsey, F.L., and Schafer, D. W. (2002), *The Statistical Sleuth* (2nd ed.), Pacific Grove CA: Duxbury.

R Core Team (2016), R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Schwarz, Gideon E. (1978), “Estimating the Dimension of a Model,” [*Annals of Statistics*](https://en.wikipedia.org/wiki/Annals_of_Statistics), 6, 461–464.

Venables, W. N. and Ripley, B. D. (2002), *Modern Applied Statistics with S (4th Ed.)*, New York: Springer-Verlag

Weisberg, S. (2014), *Applied Linear Regression* (4th edition), New Jersey: Wiley

Yeh, I-Chengh (1998), “Modeling of strength of high performance concrete using artificial neural networks”, *Cement and Concrete Research* 28(12): 1797-1808.

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